Advances in Thermal Processes and Energy Transformation
Volume 3, 2020, Strb01, pp. 75-80
ISSN 2585-9102

Design of the Minimal Length Nozzle for Ejector Cooling System by Using Method of Characteristics

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Category: Original Scientific Paper
Received : 21 October 2020 / Revised: 29 October 2020 / Accepted:30 October 2020

Keywords: ejector, ejector cooling system (ECS), minimal length nozzle (MLN), method of characteristics (MOC)

Abstract: In this paper will be presented design of the minimal length nozzle (MLN) used for ejector cooling system (ECS). Working fluid in this circuit is water (steam). Correct and precious design of the nozzle is crucial for the right operation of the ECS system. For design of MLN was used method of characteristics (MOC), which was explained in the article. In the end there was shown program made for calculation of the MLN in Graphical User Interface (GUI) of open source program Scilab.

Citation: Štrba Dominik, Mlynár Peter, Masaryk Michal, Manganos Georgios: Design of the Minimal Length Nozzle for Ejector Cooling System by Using Method of Characteristics, Advance in Thermal Processes and Energy Transformation, Volume 3, No.3, (2020), p. 75-80, ISSN 2585-9102

1 Introduction

Research of our team is focused on the ejector cooling systems (ECS) in combination with Fresnel solar collector. The ECS system runs on the heat. As working fluid was used water. Advantages of the water as a fluid are that water is cheap and available. It is also easy to work with it and it is not dangerous for human health. Another advantage of water is ecological aspect, that means water has no impact on environment, its impact on ozone layer is none (ODP=0) and also has no impact on global warming (GWP=0) [1]. Main disadvantage is that if evaporating temperature in evaporator is set for t_e=3°C, absolute pressure for liquid water to evaporate is p_e=758Pa, which is deep vacuum. For this reason is needed to create really precious nozzle which can ensure this level of the absolute pressure. In the past were used conical nozzles, which do not take in action Prandtl-Meyer expansion. The flow is not regulated for elimination of radial component of the velocity. Price and manufacturability is the reason why was this type used mostly in the past [2]. In this article will be shown how to design parabolic nozzle which take into account Prandtl-Meyer expansion.

2 Theoretical background

2.1 Ejector cooling system (ECS)

ECS creates vacuum in ejector which is non-moving part using heat for operation. Primary steam is created in the generator.

Nomenclature

A	starting point
COP	coefficient of performance
D	diameter [mm]
ECS	ejector cooling system
GWP	global warming potential
M	Mach number [-]
MLN	minimal length nozzle
MOC	method of characteristics
a	speed of sound [m/s]
n	number of characteristic lines [pcs]
p	pressure [Pa]
u	velocity [m/s]
v	velocity [m/s]
t	temperature [°C]
Greek symbols	
α	angle [°]
β	angle [°]
θ	inclination angle [°]
κ	isentropic coefficient [-]
μ	expansion angle [°]
ν	Prandtl-Meyer angle [°]
Subscripts	
1,2,3	points of the grid
e	exit of the nozzle
*	critical section

A primary steam enters into the ejector, where is expanded into higher velocity because of the convergent part of the nozzle, in place of the critical diameter *D** of the nozzle reached the Mach number, where *M*=1*, after critical diameter there is divergent part of the nozzle, where is flow accelerated even more. This accelerated flow gets out of nozzle with supersonic velocity, vacuum and suction effect are obtained for secondary steam from evaporator. In the suction chamber are primary and secondary flow sucked and in mixing chamber are mixed. In the exit of the diffusor mixed flow get out of ejector and enters into the condenser with needed condensing pressure for condensation of the steam to the liquid. From condenser is working fluid divided into two streams, first is coming back to the evaporator through expansion valve and second one is going to the generator where is primary steam created.

In comparison with a compressor cooling system, the ECS system does not have any moving or rotating parts needed for compression of the refrigerant. In the ECS lubrication is not needed because the pressure increase is made by a stationary ejector. Also there are no problems with compression in high temperature conditions. One of the biggest advantages of these systems is that when energy is most needed for cooling there is the greatest amount of solar energy. Electricity consumption is in this case lower than in the compression cooling system because here electricity is used only for pumps and they consume only about 5-10 % of the electricity used by a compressor. It means that in the ejector cooling system cooling capacity is created by the ejector and a pump unlike in the compressor cooling system, where only a compressor is used, with high electricity consumption. Another advantage of the ejector cooling system is that is a renewable source of energy [1].

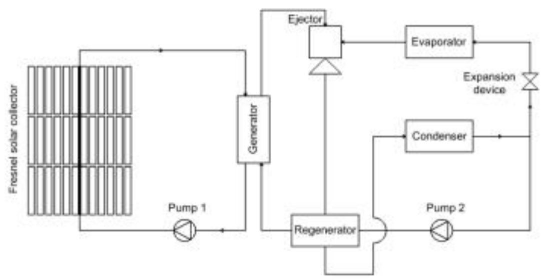

Figure 1 Ejector cooling system with Fresnel solar collector

The main disadvantage of the ECS is a low value of COP coefficient (Coefficient of performance). For this reason solar heat or waste heat is used for driving this system. It's not economical to produce heat directly for creating steam to power the cooling system [3].

2.2 Method of characteristics (MOC)

This method is used for finding geometry of the divergent part of the sonic nozzle with minimal length (MLN). Consider steady, two-dimensional, adiabatic, irrotational supersonic flow. Replace the two-dimensional current field with a rectangular grid [4, 5]. At these points, the parameters are either known, given the geometry or are subject to numerical calculation. If there are no particles in the current field that perform a vortex motion than ($v \times \nabla = 0$), velocity could be written in the form of a scalar field:

$$\bar{v} = \nabla \phi \quad (1)$$

Differential equation of continuity valid for steady state flow:

$$\nabla(\rho\bar{v}) = \frac{\partial}{\partial x}(\rho\phi_x) + \frac{\partial}{\partial y}(\rho\phi_y) = 0 \qquad (2)$$

$$\rho(\phi_{xx} + \phi_{yy}) + \phi_x \frac{\partial\rho}{\partial x} + \phi_y \frac{\partial\rho}{\partial y} = 0 \qquad (3)$$

The law of conservation of momentum in differential form is as follows:

$$dp = -\rho \frac{d(\phi_x^2 + \phi_y^2)}{2} \qquad (4)$$

The change in the density of the medium in the case of isentropic flow can be described by a change in the pressure and speed of sound:

$$dp = \frac{dp}{a^2} - \frac{\rho}{a^2} d\left(\frac{\phi_x^2}{2} + \frac{\phi_y^2}{2}\right) \qquad (5)$$

After expressing the density gradient in the x and y directions, the partial differential equation were substitute into the relation and replace it with the velocity components u and v of the partial derivatives of the scalar fields, equation then is:

$$\left(1 - \frac{u^2}{a^2}\right)\frac{\partial^2\phi}{\partial x^2} - \frac{2uv}{a^2}\frac{\partial^2\phi}{\partial x\partial y} + \left(1 - \frac{v^2}{a^2}\right)\frac{\partial^2\phi}{\partial y^2} = 0 \qquad (6)$$

If $u = \frac{\partial\emptyset}{\partial x}$ and $v = \frac{\partial\emptyset}{\partial y}$, after derivation there is

$$\mathrm{du} = d\left(\frac{\partial\phi}{\partial x}\right) = \frac{\partial^2\phi}{\partial x^2}dx + \frac{\partial^2\phi}{\partial x\partial y}dy \qquad (7)$$

$$\mathrm{dv} = d\left(\frac{\partial\phi}{\partial y}\right) = \frac{\partial^2\phi}{\partial y^2}dy + \frac{\partial^2\phi}{\partial x\partial y}dx \qquad (8)$$

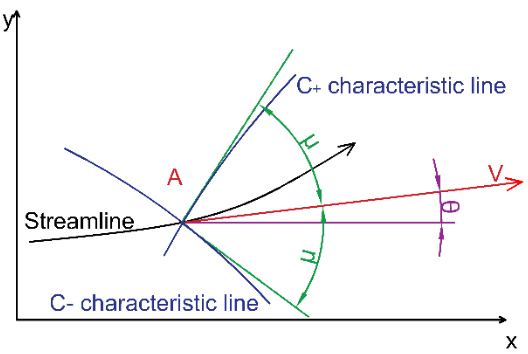

Figure 2 Illustration of left-running and right-running characteristic lines

In equations (6) – (8) there is 3 unknown, system of 3 unknown is calculated by Cramer's rule.

$$\frac{\partial^2\phi}{\partial x\partial y}dx = \frac{\begin{vmatrix} 1-\frac{u^2}{a^2} & 0 & 1-\frac{v^2}{a^2} \\ dx & du & 0 \\ 0 & dv & dy \end{vmatrix}}{\begin{vmatrix} 1-\frac{u^2}{a^2} & \frac{-2uv}{a^2} & 1-\frac{v^2}{a^2} \\ dx & dy & 0 \\ 0 & dx & dy \end{vmatrix}} \qquad (9)$$

A graphical interpretation of matrix from Equation (9) is show on the Figure 2. Point A is nodal point of the grid of flow field. If the ratio *dx/dy* is correctly chosen, determinant of denominator could be equal to zero. It means in the flow field exists characteristic line, after which the left side of the Equation (6) will be indefinite. According to [6] is this direction described with this equation:

$$\left(\frac{dx}{dy}\right)_{char} = \tan(\theta \pm \mu) \qquad (10)$$

This characteristic lines are equivalent of expanse lines arising from super and hypersonic flow. For simplicity, these lines are considered for straight lines.

Characteristic lines could be 2 types lying at an angle:

1. $\boldsymbol{\mu - \theta}$ are left-running characteristics, where $\boldsymbol{\theta + \nu(M) = const}$.
2. $\boldsymbol{\mu + \theta}$ are right-running characteristics, where $\boldsymbol{\theta - \nu(M) = const}$.

If there is known any of the parameters $\boldsymbol{\theta}, \boldsymbol{\nu}, M$ of starting point is possible to calculate every point of the grid using compatibily equations [6 - 9]. Starting point is place, where flow reached the sonic speed and its Mach number is equal to 1. This point is called critical and its diameter is called critical diameter D*.

3 Computation of the geometry of the minimal length nozzle (MLN)

Points of the grid are divided into the two groups, depends on the position and the method of finding:

1. Internal points of flow field
2. Wall points

Characteristic lines are divided into 3 groups:

1. Left-running characteristics from starting point (where Mach number is M=1)
2. Characteristic lines reflected from the axis
3. Characteristic lines captured by the wall

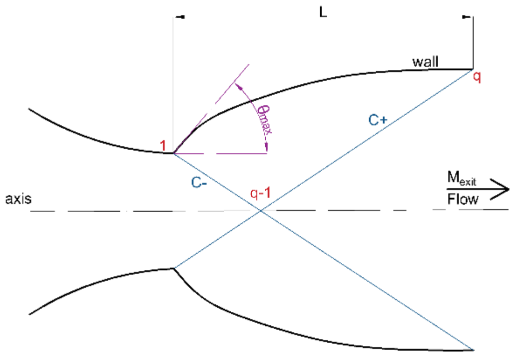

Figure 3 Contour of the MLN

Nozzle is axis symmetrical. Right-running and left-running characteristics starts from starting point, reflected from the axis and captured in the wall. Calculated grid is defined by the number and position of the intersections created during interaction of two characteristic lines. Every point is clearly characterized by θ and ν(M).

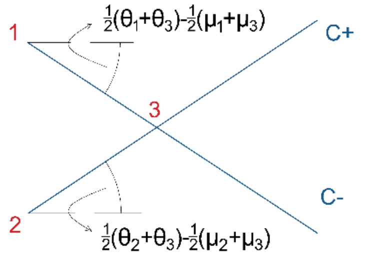

Figure 4 Approximation of characteristics by straight lines

Inclination angle of the first left-running characteristic is the first step for calculation of final geometry of the MLN. According to Prandtl-Meyer expansion theory, inclination angle in point 1 is equal to:

$$\theta_{max} = \frac{\nu_e}{2} \qquad (11)$$

Point 2 is located on this left-running characteristic and also on the axis of the nozzle, inclination angle of this point is equal to 0°.

Point 3 is located on the intersection of two characteristics. Coordinates of this point are calculated by using these equations:

$$x_i = \frac{(y_1 - x_1 \tan\alpha) - (y_1 - 1 - x_{i-1}\tan\beta)}{\tan\beta - \tan\alpha} \qquad (12)$$

$$y_i = y_{i-1} + (x_i - x_{i-1})\tan\beta \qquad (13)$$

$$\alpha = \frac{(\theta-\mu)_1 + (\theta-\mu)_i}{2} \qquad (14)$$

$$\beta = \frac{(\theta+\mu)_1 + (\theta+\mu)_i}{2} \qquad (15)$$

Mach number for the points of the grid are calculated by Prandtl-Meyer equation [7, 10]:

$$\nu(M) = \sqrt{\frac{\kappa+1}{\kappa-1}}\, atan\sqrt{\frac{\kappa+1}{\kappa-1}(M^2-1)}$$

$$- atan\sqrt{(M^2-1)} \qquad (16)$$

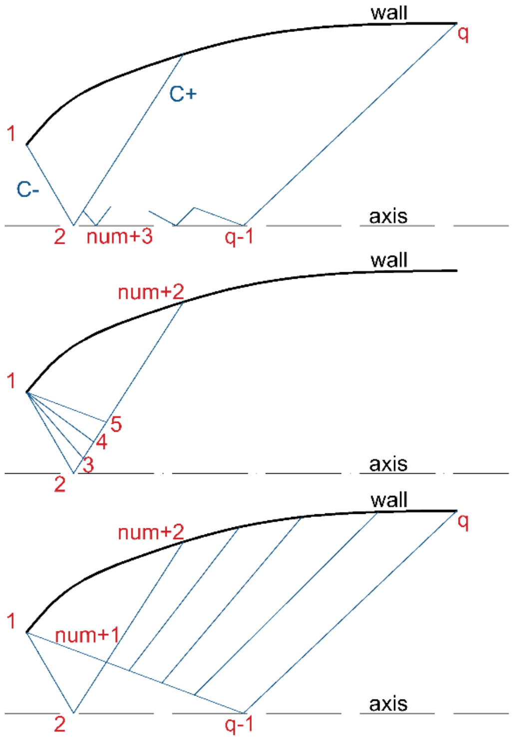

Figure 5 Different kinds of characteristic lines and points of the net

Vectors of the velocities in points which are located on the axis has only axial component, inclination angles of these points are equal to 0°. Coordinates of these points can be calculated as follow:

$$x_{num+3} = x_a \frac{y_{num+3} - y_a}{\tan\alpha} \qquad (17)$$

$$\alpha = \frac{(\theta-\mu)_a + (\theta-\mu)_{num+3}}{2} \qquad (18)$$

Internal points are created as intersections of right-running and left-running characteristics.

Wall points are used for final design of the supersonic minimal length nozzle. If there is more of the characteristic lines which starts from starting point [0,R*], the difference of the angle dθ is smaller and final contour of the nozzle is smoother. Coordinates of these points can be calculated as follow:

$$x_{num+2} = \frac{(y_1 - x_1 \tan\alpha) - (y_{num+1} - x_{num+1} \tan\beta)}{\tan\beta - \tan\alpha} \quad (19)$$

$$y_{num+2} = y_{num+1} + (x_{num+3} - x_{num+1}) \tan\beta \quad (20)$$

$$\alpha = \frac{\theta_1 + \theta_{num+2}}{2} \quad (21)$$

$$\beta = \frac{(\theta+\mu)_{num+1} + (\theta+\mu)_{num+2}}{2} \quad (22)$$

4 Program for calculation of geometry of the minimal length nozzle (MLN)

Equations from Chapter 3 where used for creating program for calculation of the contour of the MLN. This program was made in the Graphical User Interface (GUI) of the open source program Scilab. Program was made for steam as working medium. Main window and inputs are shown on Figure 6. Inputs needed for calculation of the final contour are:

- Inlet pressure *p_{in} [kPa]*
- Outlet pressure *p_{ex} [kPa]*
- Amount of the charact. lines *n [pcs]*
- Critical diameter of the nozzle *D* [mm]*
- Isentropic coefficient *κ[-]*

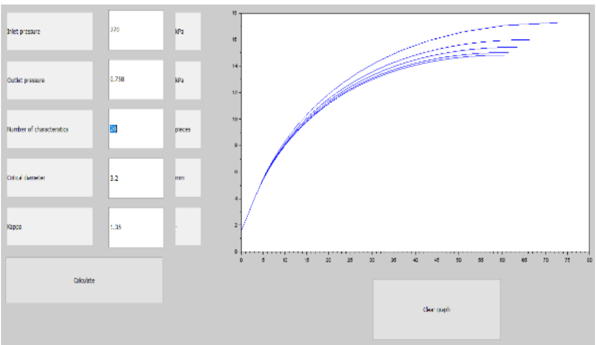

Figure 6 Graphical interface of the scilab program

As the output from the program there are [x, y, z] coordinates which can be used for creating the 3D model of the minimal length nozzle. As it shown in the Table 1, choice of the number of the characteristic lines is crucial. With increasing of number of characteristic lines, results are more precise but time needed for calculation increases exponential. For this reason, for two dimensional analysis is recommended to use minimal 50 characteristic lines.

Table 1 2D coordinates of the MLN as results from the program

No. of char.lines	x [mm]	y [mm]
5	109,09	24,97
10	82,81	19,35
20	72,67	17,27
30	68,86	16,50
40	66,33	15,99
50	64,62	15,65
60	63,52	15,43
70	62,62	15,25
80	61,52	15,03
90	61,44	15,02

With the results from the program was made 3D model of the MLN in program Autodesk Fusion 360, see Figure 7.

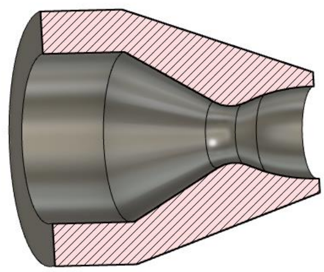

Figure 7 3D design of the minimal length nozzle

5 Conclusion

In this article was shown method for calculation contour of minimal length nozzle (MLN) used for ejector cooling system with steam as working fluid. For design of nozzle contour was used Method of Characteristics (MOC) in Graphical User Interface (GUI) in open source program Scilab. Next steps in research are to run the simulation for verifying the result from this calculation and after simulation to make experimental tests of the designed MLN.

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